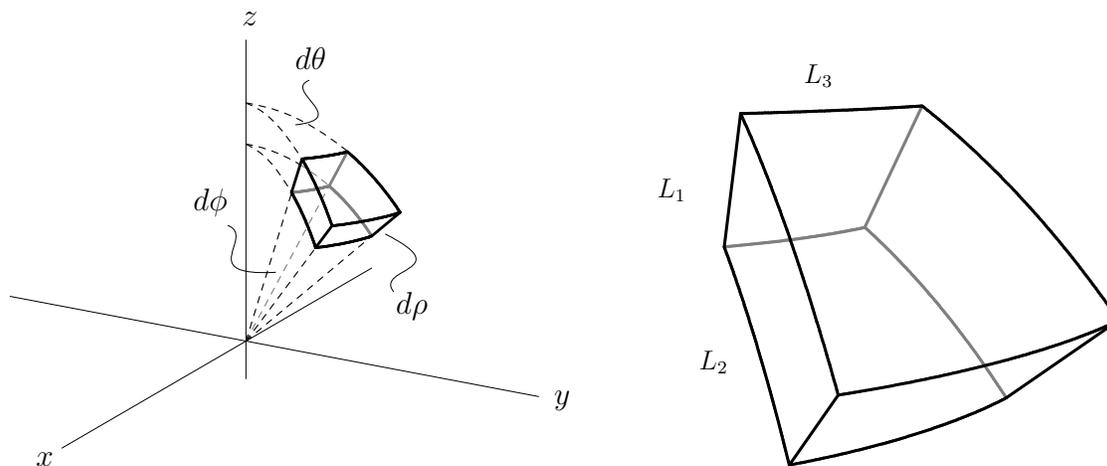


### The volume element in spherical coordinates

The figure below on the left shows a generic spherical “box” defined as the points with spherical coordinates ranging in intervals of extent  $d\rho$ ,  $d\phi$ , and  $d\theta$ . Let  $(\rho, \phi, \theta)$  be the spherical coordinates of some particular point in the box. The figure on the right shows a zoomed-in view of the box with the edge lengths labeled  $L_1$ ,  $L_2$ , and  $L_3$ .



1. What is  $L_1$  in terms of  $\rho$ ,  $\phi$ ,  $\theta$ ,  $d\rho$ ,  $d\phi$ , and  $d\theta$  (as needed)?
2. The length  $L_2$  is the length of an arc of a circle.
  - (a) What is the radius of this circle?
  - (b) What is the angle subtended by the arc?
  - (c) What is  $L_2$  in terms of  $\rho$ ,  $\phi$ ,  $\theta$ ,  $d\rho$ ,  $d\phi$ , and  $d\theta$  (as needed)?
3. The length  $L_3$  is also the length of an arc of a circle.
  - (a) What is the radius of this circle?  
Hint: Draw a vertical cross-section of the sphere.
  - (b) What is the angle subtended by the arc?
  - (c) What is  $L_3$  in terms of  $\rho$ ,  $\phi$ ,  $\theta$ ,  $d\rho$ ,  $d\phi$ , and  $d\theta$  (as needed)?
4. Let  $dV$  be the volume of the box. What is  $dV$  in terms of  $\rho$ ,  $\phi$ ,  $\theta$ ,  $d\rho$ ,  $d\phi$ , and  $d\theta$  (as needed)?